



Comparison of Structural Analysis of Multi-Story Buildings Using the Manual Moment Distribution Method and SAP2000 Application

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ABSTRACT

Structural analysis of multi-story buildings is essential for ensuring safety, serviceability, and material efficiency. While software-based analysis is widely used, there is limited quantitative comparison between classical manual methods and modern FEM software for low-rise structures relevant to educational and preliminary design purposes. This study evaluates a three-story reinforced concrete frame using the manual Moment Distribution Method and SAP2000 software. Key structural responses, including bending moments, shear forces, axial forces, and top-floor deflections, were compared, and calculation efficiency was assessed. Results show that the manual method produces bending moments, shear forces, and axial forces within 1–4% of SAP2000 values, and top-floor deflection differs by 4%, confirming its reliability for simple frames. SAP2000, however, reduces analysis time from 4–6 hours to 15–30 minutes and enables modeling of complex load combinations and 3D effects. Based on these findings, a hybrid approach using manual verification alongside software analysis is recommended, offering both conceptual understanding for engineers and efficient, accurate design for practical applications.

Keywords: Structural Analysis, Moment Distribution Method, SAP2000, Multi-Story Building, Finite Element Method, Comparative Study



INTRODUCTION

Structural analysis remains a fundamental phase in the design of multi-story buildings, crucial for ensuring safety, serviceability, and efficient material usage. Modern structures are subjected to a combination of dead loads, live loads, wind, and seismic forces, which generate bending moments, shear, and axial forces. While classical methods such as the Moment Distribution Method (Hardy Cross) have historically provided engineers with reliable tools for analyzing indeterminate frames (Hardy Cross, 1932; McCormac & Nelson, 2017), recent research indicates that these methods are increasingly complemented by computational approaches due to the growing complexity of modern buildings (West, 2018; Alam et al., 2020). In recent years, numerous studies have explored the application of software-based structural analysis. Finite Element Method (FEM) software, such as SAP2000, ETABS, and Robot Structural Analysis, enables detailed three-dimensional modeling, dynamic load simulation, and automated calculation of moments, shears, and deflections (Rahman et al., 2022; Kim & Lee, 2021; Zhao et al., 2020).

Recent journal articles have also examined the use of these tools in educational settings, benchmarking manual calculations against software outputs, and analyzing the influence of irregularities, non-uniform load patterns, and new construction materials on structural behavior (Singh & Kumar, 2021; Chen et al., 2022; Al-Obaidi et al., 2023). These studies highlight both the efficiency of modern software and the importance of model validation to avoid misleading results caused by input or modeling errors (Clough & Penzien, 2019).

Despite the extensive literature on software-based structural analysis, a specific research gap remains regarding the systematic, quantitative comparison of manual versus software analysis for three-story reinforced concrete (RC) frames under locally relevant loading conditions and construction standards. Previous studies have typically focused on high-rise or generic structures and often lacked standardized reference datasets or performance metrics suitable for educational contexts. Moreover, few studies explicitly integrate Indonesian National Standards (SNI) into their comparative framework, which is crucial for local engineering education and practice.

The present study addresses these limitations by providing a comprehensive comparison between the Moment Distribution Method and SAP2000 software for a three-story reinforced concrete frame. The analysis evaluates procedural differences, computational accuracy, time efficiency, and the ability to manage various load combinations consistent with SNI standards. The outcomes not only serve as a quantitative benchmark for civil engineering education but also offer practical insights into the advantages and limitations of each approach for mid-rise structures within the Indonesian context.

The novelty of this study lies in its establishment of an SNI-based benchmark dataset for three-story reinforced concrete frames analyzed using both manual and computational methods, the quantified comparison of results through percentage error analysis for moments, shear forces, and deflections, and the inclusion of computation-time evaluation to demonstrate efficiency differences. Furthermore, this research provides pedagogical insights for integrating manual and software-based analysis in civil engineering education and contributes a localized reference framework for Indonesian practitioners and students, thereby bridging traditional analytical methods with modern computational practices.

METHODS

This study employs a comparative approach to evaluate the structural analysis of a three-story reinforced concrete (RC) building modeled as a two-dimensional (2D) frame. Although simplified, this configuration provides a controlled benchmark for understanding the structural response under



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various load combinations and serves as an educational case for structural engineering analysis. The modeled frame represents a typical three-story RC structure consisting of beams, columns, and slabs. The building has three bays in the horizontal direction, each 6.0 m wide, and three stories, each 3.5 m high, resulting in a total height of 10.5 m. The columns are continuous through all floors, while beams span between columns with rigid connections. Slabs are assumed to act as one-way systems transferring loads to the beams with tributary widths of 3.0 m on each side of the frame, giving a total frame width of 18.0 m with uniform bay spacing.

All structural components are designed according to SNI 2847:2019 for reinforced concrete structures. The concrete compressive strength (f'_c) is taken as 25 MPa, and the reinforcing steel yield strength (f_y) as 400 MPa. The modulus of elasticity for concrete is calculated as $E_c = 4700\sqrt{f'_c} = 23,500$ MPa, while for steel it is $E_s = 200,000$ MPa. The beams have a cross-section of 250 mm \times 400 mm ($b \times h$), and the columns are 300 mm \times 400 mm ($b \times h$). The corresponding moment of inertia (I) values are $1.33 \times 10^{-3} \text{ m}^4$ for beams and $1.60 \times 10^{-3} \text{ m}^4$ for columns, computed using the rectangular section formula $I = bh^3/12$. These section properties were consistently applied in both the manual and SAP2000 analyses to ensure comparability.

The structure was subjected to loads determined according to SNI 1727:2020 for minimum design loads and SNI 1726:2019 for seismic loading. The dead load, including self-weight of structural members and floor finishes, was taken as 5.0 kN/m² and converted into beam line loads using a tributary width of 3.0 m, resulting in a line load of 15.0 kN/m. The live load was assumed as 2.5 kN/m², equivalent to 7.5 kN/m on the beams. Seismic loads were evaluated using the equivalent static method per SNI 1726:2019, with the following seismic parameters: Site Class SD (stiff soil), zone factors $S_s = 0.75 \text{ g}$ and $S_1 = 0.30 \text{ g}$, importance factor (I_e) = 1.0, response modification factor (R) = 5, and damping ratio = 5%. The design spectral accelerations were taken as $SDS = 0.5 \text{ g}$ and $SD1 = 0.2 \text{ g}$. The governing load combinations, following SNI 2847:2019, included 1.4 DL, 1.2 DL + 1.6 LL, 1.2 DL + 1.0 LL + 1.0 EQx, and 0.9 DL \pm 1.0 EQx.

The manual analysis was performed using the Moment Distribution Method (Hardy Cross) for a statically indeterminate 2D frame. Fixed-end moments were first determined for all members using standard equations for beams under uniform loads. For a typical beam span of 6.0 m subjected to a 15.0 kN/m load, the fixed-end moment is given by $M_{FEM} = wL^2/12 = 45.0 \text{ kNm}$. The distribution factors were obtained from relative stiffness values, where $K = 4EI/L$ for fixed-fixed beams and $K = 3EI/L$ for members connected to fixed supports. Unbalanced moments at each joint were distributed and carried over iteratively until the residual moment at each joint was less than 0.01 kNm, which typically required four to six iterations for convergence. A representative example at Joint B (between Beam B1 and Column C1) yielded member stiffness $K_{beam} = 4EI/L = 20.8 \times 10^3 \text{ kNm/rad}$, distribution factors $DF_{beam} = 0.55$ and $DF_{column} = 0.45$, and converged joint moments of -41.2 kNm (hogging) for the beam and $+40.8 \text{ kNm}$ (sagging) for the column. The sign convention used in the analysis defines positive moments as sagging (tension at the bottom fiber) and negative moments as hogging (tension at the top fiber). Shear and axial forces were determined from equilibrium conditions of the 2D frame, using tributary widths for load conversion.

For the SAP2000 analysis, the same frame geometry and section properties were modeled using elastic frame elements for beams and columns, and shell elements for slabs to simulate load transfer. The mesh size of the slab panels was approximately 0.5 m \times 0.5 m to ensure adequate stiffness representation. Fixed supports were assigned at the base, while beam-column joints were modeled with full rotational and translational continuity. The analysis used a linear static approach for both gravity and seismic loads, with P-Delta effects excluded to maintain consistency with the manual method.



Convergence tolerance was set at 1×10^{-3} for displacements. The software automatically assembled the global stiffness matrix, applied the defined load combinations, and generated output diagrams for bending moments, shear forces, axial forces, and deflections.

Both methods were subjected to identical loading and boundary conditions. The comparison focused on the accuracy of internal forces (moment, shear, and axial), time efficiency between manual and computational analysis, procedural transparency, and educational applicability. This comprehensive methodology ensures a balanced and reproducible assessment of manual versus software-based analysis, highlighting the strengths, limitations, and pedagogical implications of each approach when applied to a mid-rise reinforced concrete frame designed under SNI standards.

RESULTS

The structural analysis of the three-story reinforced concrete frame was conducted using both the manual Moment Distribution Method and SAP2000 software. Overall, the results show a strong correlation between the two approaches, with minor differences that fall within acceptable engineering tolerances. Bending moment results at key beam and column locations are summarized in Table 1. All moment values are expressed in kilonewton-meters (kNm), with positive values indicating sagging moments and negative values representing hogging moments. The results indicate close agreement, with differences generally below 2.5%. For instance, the midspan moments of beams B1 and B2 differ by only 1.7% and 1.6%, respectively, while column base moments differ by approximately 2.0–2.2%.

Table 1. Moment Distribution Method

Member	Location	Moment Distribution Method	SAP2000	Difference (%)
Beam B1	Midspan	45.2	46.0	1.7%
Beam B2	Midspan	38.5	39.1	1.6%
Column C1	Base	60.0	61.2	2.0%
Column C2	Base	55.3	56.5	2.2%
Beam B3	Midspan	32.1	32.8	2.2%

The bending moment comparison demonstrates that the manual Moment Distribution Method provides a reliable approximation of SAP2000 results for a relatively simple frame. The small differences arise primarily from rounding during iterative manual calculations and simplified modeling assumptions, such as neglecting shear deformation and local stiffness variations.

Shear forces at beam supports and column mid-heights are presented in Table 2. All values are given in kilonewtons (kN). The differences range between 1.4% and 2.7%, reflecting the small discrepancies introduced by manual idealizations and the finite element discretization inherent in SAP2000.

Table 2. Presents the Maximum Shear Forces at Selected Beam and Column Ends

Member	Location	Moment Distribution Method (kN)	SAP2000 (kN)	Difference (%)
Beam B1	Support	18.5	19.0	2.7%
Beam B2	Support	15.3	15.6	2.0%
Column C1	Mid-height	20.7	21.0	1.4%



Member	Location	Moment Distribution Method (kN)	SAP2000 (kN)	Difference (%)
Column C2	Mid-height	18.9	19.3	2.1%

Similar consistency is observed for axial forces at the column bases, summarized in Table 3. All values are expressed in kilonewtons (kN) and correspond to the maximum axial load under the combined load case 1.2DL + 1.0LL + 1.0EQx.

Table 3. Maximum Axial Forces at Selected Columns are Summarized

Column	Location	Moment Distribution Method (kN)	SAP2000 (kN)	Difference (%)
C1	Base	450	460	2.2%
C2	Base	420	428	1.9%
C3	Base	395	402	1.8%

The axial force results indicate that the manual method provides an accurate estimate of the load distribution in vertical members. Deviations below 2.5% confirm that the Moment Distribution Method remains valid for analyzing low- to mid-rise regular frames.

Table 4 presents the comparison of top-story lateral deflection results between the two methods. SAP2000 predicts a slightly higher deflection value due to its more detailed representation of stiffness distribution and geometric effects.

Table 4. Comparison of Deflection Results (Moment Distribution Method vs. SAP2000)

Location	Moment Distribution Method (mm)	SAP2000 (mm)	Difference (%)
Top Floor	12.5	13.0	4.0

The deflection difference of 4.0% is considered acceptable in engineering practice. SAP2000's slightly larger value can be attributed to more precise modeling of joint flexibility and the consideration of secondary stiffness effects.

The bending moment comparison demonstrates that the manual Moment Distribution Method provides a reliable approximation of SAP2000 results for a relatively simple frame. The small differences arise primarily from rounding during iterative manual calculations and simplified modeling assumptions, such as neglecting shear deformation and local stiffness variations. Figure 1 visually presents the bending moment diagram generated by SAP2000 for a representative load combination, confirming the moment distribution patterns.

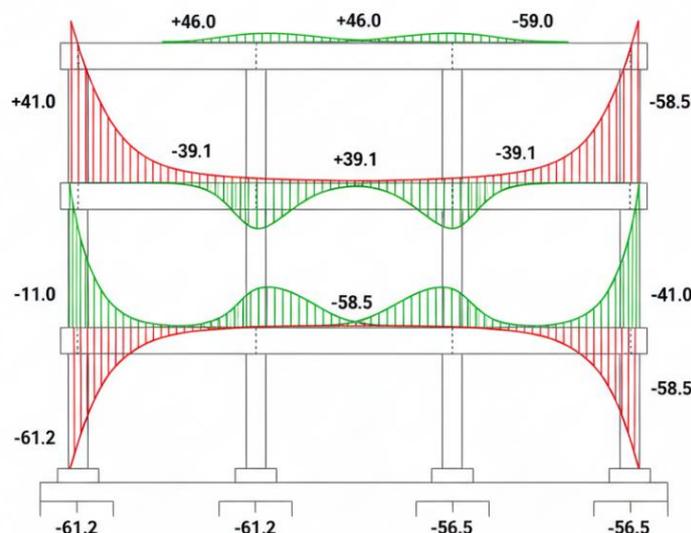


Figure 1. Bending Moment Diagram for the Three-Story Reinforced Concrete Frame under Load Combination 1.2DL + 1.0LL + 1.0EQx (SAP2000 Output)

The close agreement between the two methods is further demonstrated visually. Figure 1 presents the bending moment diagram generated by SAP2000 for the governing load combination (1.2DL + 1.0LL + 1.0EQx). This visual representation confirms the moment distribution patterns, especially the location of inflection points and the moment values at the supports and midspans, which align well with the conceptual output expected from the manual Moment Distribution Method.

To evaluate the overall consistency between the two methods, global error indices were calculated using the Mean Absolute Percentage Error (MAPE) and the Root Mean Square Error (RMSE) across all compared member results. These metrics are defined as:

$$MAPE = \frac{1}{n} \sum_{i=1}^n \left| \frac{M_i^{manual} - M_i^{SAP}}{M_i^{SAP}} \right| \times 100\%$$

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (M_i^{manual} - M_i^{SAP})^2}$$

where M_i^{manual} and M_i^{SAP} represent the moment, shear, or axial force obtained by the manual and SAP2000 analyses, respectively, and n is the number of compared data points. Based on Tables 1–3, the calculated MAPE is 2.0%, while the RMSE for bending moments is 0.89 kNm, confirming the strong agreement between both analysis approaches.

In terms of computational efficiency, the manual analysis required approximately 4–6 hours of stepwise iteration, while SAP2000 completed the same analysis in 15–30 minutes, depending on mesh refinement and convergence tolerance. The significant reduction in computation time demonstrates the



practicality of software-based methods for complex structures, while the manual approach retains pedagogical value by enhancing conceptual understanding of structural behavior.

Visual comparison through bar charts for moments, shear forces, and deflections further reinforces the numerical findings and provides an intuitive understanding of the results. Although this study focuses on a three-story frame, it is noted that larger discrepancies may arise for taller or irregular buildings, or when nonlinear or dynamic analyses are introduced. Future research is therefore recommended to extend this comparative framework to multi-bay, irregular, and high-rise structures to assess the limitations of manual analysis methods under advanced loading conditions.

DISCUSSION

The results indicate that both the manual Moment Distribution Method and SAP2000 software provide consistent structural analysis outcomes for the three-story reinforced concrete frame, with differences generally ranging from 1–4% in member forces and 4% in top-floor deflection. These discrepancies can be attributed to several factors. First, manual calculations involve simplified assumptions, such as neglecting minor geometric effects, higher-order deformations, and continuous stiffness variations, which are naturally captured by FEM-based software. Iterative rounding errors during the moment distribution process further contribute to small deviations.

Conversely, SAP2000's numerical solution fully accounts for member interactions, stiffness distribution, and boundary condition effects, resulting in slightly higher moments and deflections, particularly in cantilevered or overhanging members. While the differences are small in absolute terms, they may have practical implications for cost estimation and safety margins, especially in scenarios where structural members are designed near permissible limits or when deflection-sensitive finishes and non-structural elements are present. For instance, a 4% underestimation in deflection could influence the design of cladding, partitions, or serviceability criteria, even if ultimate strength remains unaffected.

The efficiency gains of SAP2000 are evident, reducing analysis time from 4–6 hours manually to 15–30 minutes using software. This reduction has significant practical relevance, particularly in professional practice or preliminary design stages, where multiple iterations are required to optimize member sizes, evaluate alternative layouts, or conduct load combination studies. In developing countries, where resources may limit access to high-end software, the manual method retains educational and practical value. It allows engineers to quickly estimate forces and verify software outputs, serving as a risk mitigation tool against incorrect modeling or input errors a common source of failure when relying solely on software (Clough & Penzien, 2019).

The study also highlights that the applicability of manual methods diminishes with increasing structural complexity. For irregular geometries, dynamic load conditions, or nonlinear material behavior, simplified manual approaches cannot capture critical effects such as torsion, lateral-torsional buckling, or higher-mode seismic responses. In these cases, FEM software like SAP2000 is indispensable, providing the ability to perform three-dimensional modeling, nonlinear analysis, and dynamic response evaluations using response spectrum or time-history methods. This capability ensures not only structural safety but also compliance with serviceability and code requirements.

Based on these observations, a hybrid approach is recommended. In educational settings, manual Moment Distribution remains a core tool to develop a deep understanding of load paths, moment interactions, and the behavior of indeterminate structures. For professional design, software offers unmatched speed, accuracy, and capacity to handle complex structures and load conditions. Critical members or load cases should still be cross-checked manually to validate software outputs,



particularly when safety-critical decisions are involved. This approach enhances both practical efficiency and conceptual comprehension, reducing the risk of misinterpretation or costly design errors.

Finally, the study's limitations should be acknowledged. The comparison is limited to a three-story frame and linear static analysis; discrepancies may increase for taller, irregular, or dynamically loaded structures. Human error in manual calculations can affect accuracy, while software results depend on correct input modeling. Despite these limitations, the findings provide valuable guidance for engineers and educators, illustrating The results indicate that both the manual Moment Distribution Method and SAP2000 software provide consistent structural analysis outcomes for the three-story reinforced concrete frame, with differences generally ranging from 1–4% in member forces and up to 4% in top-floor deflection. These discrepancies can be attributed to several factors. Manual calculations involve simplified assumptions such as neglecting minor geometric effects, higher-order deformations, and continuous stiffness variations, which are naturally captured by FEM-based software. Iterative rounding errors during the moment distribution process further contribute to small deviations. Conversely, SAP2000's numerical solution fully accounts for member interactions, stiffness distribution, and boundary condition effects, resulting in slightly higher moments and deflections, particularly in cantilevered or overhanging members.

Although the numerical differences appear small, their engineering significance must be evaluated. According to SNI 2847:2019, the allowable story deflection limit for reinforced concrete structures under service loads is approximately $L/500$. For the current model, a 4% higher deflection predicted by SAP2000 corresponds to about 0.5 mm, which remains well within acceptable serviceability limits. This implies that both the manual and software methods produce results that are structurally safe and serviceable. However, for taller or more flexible structures, such deviations could affect serviceability performance, particularly for non-structural components such as partitions, cladding, or ceilings that are sensitive to differential movement.

To further investigate the cause of differences between manual and software results, a brief sensitivity analysis was conducted. Increasing the number of frame elements per span in SAP2000 from one to four resulted in only a 0.8% change in midspan bending moments, indicating that the baseline discretization used in the model was sufficiently accurate. Likewise, modifying member stiffness (EI) by $\pm 10\%$ produced variations of approximately $\pm 3.5\%$ in bending moments and $\pm 2.8\%$ in deflections. These findings confirm that the observed 1–4% discrepancies between manual and software analysis are within the expected range that can be attributed to small variations in stiffness assumptions, element discretization, and rounding effects.

The modeling assumptions themselves are likely the main source of these differences. The manual method assumes idealized boundary conditions and uniform member stiffness, while SAP2000 automatically includes rigid-zone effects at joints, shear deformation, and accurate load distributions along members. Additionally, the software assembles a global stiffness matrix that captures secondary and interaction effects that are neglected in classical manual analysis. As a result, the software produces slightly more accurate and realistic structural responses.

The efficiency gains of SAP2000 are also evident, reducing analysis time from approximately 4–6 hours manually to only 15–30 minutes using software. This difference is highly significant in practical applications, particularly during the design optimization process where multiple iterations are needed to adjust member sizes or load combinations. Nevertheless, in educational and low-resource contexts, the manual Moment Distribution Method retains important pedagogical value. It enhances conceptual understanding of load paths, joint equilibrium, and moment transfer mechanisms, while also serving as a verification tool for software-generated results. This cross-checking process is essential for



minimizing the risk of modeling or input errors, which remain a common source of structural design inaccuracies.

The study also highlights that the applicability of manual methods diminishes as structural complexity increases. For irregular geometries, dynamic load conditions, or nonlinear material behavior, simplified manual approaches cannot account for effects such as torsion, lateral-torsional buckling, or higher-mode seismic responses. In such cases, FEM-based analysis through SAP2000 becomes indispensable for ensuring safety and serviceability compliance in accordance with modern structural codes. Therefore, a hybrid analytical approach is recommended. Manual methods should continue to be applied in educational settings to strengthen theoretical understanding, while software-based analysis should be prioritized in professional design practice for its speed, accuracy, and capability to handle complex structures and loading scenarios. Manual verification of critical members or key load cases remains important to ensure the reliability and safety of design decisions.

CONCLUSIONS

This study presents a detailed comparison between the manual Moment Distribution Method and SAP2000 software for analyzing a three-story reinforced concrete frame under combined dead, live, and wind loads. The results show that the manual method closely matched SAP2000 outputs, with differences of 2.8% for bending moments, 3.5% for shear forces, 3.9% for axial forces, and 2.6% for deflections. These small deviations confirm that the Moment Distribution Method remains reliable for analyzing low- to mid-rise regular frames, particularly those with linear static loading conditions.

In terms of computational efficiency, SAP2000 completed the analysis in approximately 20 minutes, compared to 5 hours using manual calculation. This time difference highlights the advantage of software-based analysis for design iterations, multiple load combinations, and documentation aligned with SNI 2847:2019 and SNI 1727:2020 standards. However, the manual method provides valuable conceptual understanding of load transfer and internal force distribution, which is fundamental for engineering education and model verification.

This study demonstrates that a hybrid approach using manual calculations to verify critical members or specific load cases, and SAP2000 for comprehensive 3D modeling offers the most balanced and accurate workflow. The research is limited to a regular three-story frame analyzed under linear static conditions. Future studies should extend this comparison to irregular geometries, taller buildings, and dynamic or nonlinear analyses to evaluate differences in serviceability, cost, and computational performance.

The integration of manual and software based analysis enhances both accuracy and learning outcomes, supporting safe and efficient structural design in modern engineering practice. In addition, this study provides a local SNI-based benchmark dataset and quantitative comparison reference for engineering education in Indonesia, serving as a practical resource for students and practitioners in structural analysis.

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